More about on the short distance contribution to the $B_c \to B_u^* \gamma$ decay

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Abstract

We calculate the transition form factor for the $B_c \to B_u^* \gamma$ decay taking into account only the short distance contribution, in framework of QCD sum rules method. We observe that the transition form factor predicted by the QCD sum rules method is approximately two times larger compared to the result predicted by the Isgur, Scora, Grinstein and Wise model.

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1 Introduction

Flavor changing neutral current (FCNC) transitions constitute one of the most important research area in particle physics. In standard model (SM) these transitions take place only at one loop level. Therefore the study of the rare decays allows us to check the gauge structure of the SM and can provide valuable information for a more precise determination of the Cabibbo–Kobayashi–Maskawa matrix elements, leptonic decay constants, etc., which are poorly known today.

In the SM the FCNC transitions of the down–quark sector have relatively large branching ratio, due to the large mass of the top quark running in the loop and $b \to s$ transition has already been observed in experiments [1]. On the other hand, in up quark–sector of the SM these transitions are quite rare since in the loop down quark runs which has smaller mass compared to top quark mass. At present only the upper experimental bounds on the FCNC transitions of the up quark sector exist [2].

To probe the very rare $c \to u\gamma$ transition in SM, it was shown in [3] that the radiative beauty conserving $B_c \to B_u^*\gamma$ decay is very promising. It should be noted that B_c meson has been observed by the CDF Collaboration at Fermilab [4]. This decay receives short and long distance contributions. The short distance contribution in $B_c \to B_u^*\gamma$ decay comes from FCNC $c \to u\gamma$ transition when $\bar{b}$ is a spectator quark. The long distance contributions to $B_c \to B_u^*\gamma$ decay can be grouped into two classes:

I) Vector meson dominance contribution which corresponds the processes $c \to uq_i\bar{q}_i$, is followed by the conversion of $q_i\bar{q}_i$ pair to photon while $\bar{b}$ is spectator again, which is similar to the short distance contribution case.

II) Annihilation contribution mechanism, which corresponds to the annihilation process $c\bar{b} \to u\bar{b}$ where photon is attached to any quark line.

The short and long distance effects to this decay are calculated in framework of the Isgur, Scora, Grinstein and Wise (ISGW) model [5] and it is found that both contributions are comparable to each other which allows, in principle, probing $c \to u\gamma$ transition. It is found that the branching ratio is of the order of $\sim 10^{-8}$ and can be detectable at future LHC. This result is quite interesting and is the first example where short and long distance effects for the $c \to u\gamma$ transition are comparable, contrary to the corresponding D meson decays for which long distance contribution is dominant [6]–[8]. Therefore this observation opens the way to extract the short distance $c \to u\gamma$ contribution from the experiment. For this reason, in order to check this principal result it is necessary to perform these calculations still in another framework.

In the present letter, we calculate the form factor for the $B_c \to B_u^*\gamma$ decay due to the short distance contribution only, in frame work of the QCD sum rules. It is observed that the value of the form factor calculated in the QCD sum rules is approximately two times larger compared to the one predicted by the ISGW model. As a result, it seems that in the $B_c \to B_u^*\gamma$ decay case, the short and long distance contributions are of the same order. This circumstance opens the way for a real possibility of probing rare $c \to u\gamma$ transition via $B_c \to B_u^*\gamma$ decay.

As has been noted already, we restrict ourselves only to the short distance contribution to the $B_c \to B_u^*\gamma$ decay. The short distance contribution to the $B_c \to B_u^*\gamma$ decay is obtained from $c \to u\gamma$ transition, where $\bar{b}$ quark is a spectator. The effective Hamiltonian for the

$c \to u\gamma$ transition is given as

$$\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}}\frac{e}{4\pi^2} V_{cs}V_{us}^* C_7(\mu)\bar{u}\sigma_{\mu\nu}\left[m_c\frac{1+\gamma_5}{2} + m_u\frac{1-\gamma_5}{2}\right] c\,\mathcal{F}^{\mu\nu} \ , \tag{1}$$

where V_{ij} correspond to the CKM matrix elements, and $\mathcal{F}_{\mu\nu}$ is the electromagnetic field strength tensor. The appropriate scale for $C_7(\mu)$ is $\mu = m_c$ since $\bar{b}$ quark is the spectator for the short distance contribution in the $B_c \to B_u^*\gamma$ decay. In further calculations we will take the mass of the up quark to be zero.

The two loop QCD corrections to the $c \to u\gamma$ transition was calculated in [9] whose prediction is $C_7(m_c) = -0.0068 - 0.02i$ and this result is scheme independent. In order to calculate the amplitude for the $B_c \to B_u^*\gamma$ decay, the matrix elements

$$\langle B_u^* \left| \bar{u}\sigma_{\mu\nu}(1\pm\gamma_5)q^\nu \right| B_c\rangle \ ,$$

need to be calculated at $q^2 = 0$, where q is the photon four–momentum. These matrix elements can be written in terms of two gauge invariant form factors $F_1(0)$ and $F_2(0)$ as follows:

$$\begin{aligned}
\langle B_u^*(p',\varepsilon') \left| \bar{u}i\sigma_{\mu\nu}q^\nu c \right| B_c(p)\rangle &= i\epsilon_{\mu\alpha\beta\rho}\varepsilon'^\alpha p'^\beta q^\rho F_1(0) \ , \\
\langle B_u^*(p',\varepsilon') \left| \bar{u}i\sigma_{\mu\nu}q^\nu\gamma_5 c \right| B_c(p)\rangle &= \left[\left(m_{B_c}^2 - m_{B_u^*}^2\right)\varepsilon'_\mu - (\varepsilon' q)\,(p+p')_\mu\right] F_2(0) \ .
\end{aligned} \tag{2}$$

Using the relation

$$\sigma_{\mu\nu}\gamma_5 = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\sigma^{\alpha\beta} \ , \tag{3}$$

one can easily show that $F_2(0) = F_1(0)/2$. Therefore in order to calculate the short distance part of the $B_c \to B_u^*\gamma$ decay it is enough to calculate $F_1(0)$ or $F_2(0)$, for which we will employ the three–point QCD sum rules [10, 11]. For the evolution of the form factor $F_1(0)$ in framework of the QCD sum rules, we consider the following three–point function

$$\Pi_{\mu\alpha} = -\int d^4x d^4y e^{i(px-p'y)} \left\langle 0 \left| \mathrm{T}\left\{\bar{b}(y)\gamma_\alpha u(y)\bar{u}(0)i\sigma_{\mu\nu}q^\nu c(0)\bar{c}(x)i\gamma_5 b(x)\right\}\right| 0\right\rangle \ , \tag{4}$$

where $\bar{b}\gamma_\alpha u$ and $\bar{c}i\gamma_5 b$ are the interpolating currents for states with the B_u^* and B_c mesons, respectively. The Lorentz structure in the correlator (4) can be written as

$$\Pi_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta\,\Pi \ , \tag{5}$$

where scalar amplitude Π is the function of the kinematical invariants, i.e., $\Pi = \Pi(p^2, p'^2)$.

In accordance with the usual QCD sum rules philosophy, the theoretical part of the three–point correlator can be calculated by employing the operator product expansion (OPE) for the T–ordered product of currents in (4). The values of the heavy quark condensates are related to the vacuum expectation values of the gluon operators. For example

$$\left\langle Q\bar{Q}\right\rangle = -\frac{1}{12m_Q}\frac{\alpha_s}{\pi}\left\langle G^2\right\rangle - \frac{1}{360m_Q^3}\frac{\alpha_s}{\pi}\left\langle G^2\right\rangle \cdots \ , \tag{6}$$

where Q is the heavy quark and the heavy quark condensate contributions are suppressed by inverse of the heavy quark mass. for this reason we safely omit them in our calculations.

It should be stressed that the light quark condensate does not give any contribution to the above–mentioned decay after double Borel transformation. Therefore the only non–perturbative contribution to the $B_c \to B_u^* \gamma$ decay comes from gluon condensate.

So, in the lowest order of perturbation theory, the three–point function is given by the bare quark loop and by gluon condensate contribution. The contribution to the three–point function from the bare loop can be obtained using the double dispersion representation in p^2 and p'^2

$$\Pi^{per}(p^2,p'^2) = -\frac{1}{4\pi^2} \int \frac{\rho^{per}(p^2,p'^2)}{(s-p^2)(s'-p'^2)} ds ds' + \text{sub. terms} \; . \tag{7}$$

The spectral density $\rho^{per}(p^2,p'^2)$ can be calculated using the Cutkovsky rule, i.e., by replacing propagators with delta functions:

$$\left(k^2 - m_i^2\right)^{-1} \to -2\pi i \delta(k^2 - m_i^2) \; . \tag{8}$$

After standard calculations for the spectral density we get

$$\rho^{per}(s,s') = 4N_c \Big\{ m_c m_b \left[A_1 + A_2 + \mathcal{I}_0\right] - m_c^2 A_1 - 2A \Big\} \; , \tag{9}$$

where

$$\begin{aligned}
A_1 &= \frac{2\mathcal{I}_0}{(s-s')^2} \left[s'(s + m_b^2 - m_c^2) - \frac{1}{2}(s+s')(s'+m_b^2) \right] , \\
A_2 &= \frac{2\mathcal{I}_0}{(s-s')^2} \left[\frac{1}{2}(s+s')(m_c^2 - m_b^2 - s) + s(s'+m_b^2) \right] , \\
A &= \mathcal{I}_0 \, \frac{m_c^2 \left[m_c^2 s' + (m_b^2 - s')(s-s')\right]}{2(s-s')^2} , \\
\mathcal{I}_0 &= -\frac{1}{4(s-s')} ,
\end{aligned} \tag{10}$$

and N_c is the color number.

The region of integration over s and s' is determined by the following inequalities

$$\begin{gathered}
m_b^2 \le s' \le s'_0 \; , \\
s' - \frac{s' m_c^2}{m_b^2 - s'} \le s \le s_0 \; .
\end{gathered} \tag{11}$$

Note that we have neglected $\mathcal{O}(\alpha_s/\pi)$ hard gluon corrections to the triangle diagram, as they are not available yet. However, we expect their contribution to be about $\sim 10\%$, so that if the accuracy of the QCD sum rules is taken into account, these corrections would not change the results drastically.

From our result on spectral density we can get the spectral density for $B \to K^*\gamma$ decay (when $m_s \to 0$) if we formally make the replacements $m_b \to 0$ and $m_c \to m_b$ in Eqs. (9) and (10). Indeed, our results coincide with the results of [12] for $B \to K^*\gamma$ decay, after the above–mentioned substitutions are performed. The gluon condensate contribution to three–point correlator (4) is given by diagrams depicted in Fig. (1). The calculations of these diagrams were carried out in the Fock–Schwinger fixed point gauge [13, 14, 15]; $x^\mu A_\mu(x) = 0$. For calculation of the gluon condensate contributions, we have used the Schwinger representation for the Euclidean propagators, i.e.,

$$\frac{1}{\left[k^2+m^2\right]^a} = \frac{1}{\Gamma(a)} \int_0^\infty d\alpha\, \alpha^{n-1} e^{-\alpha(k^2+m^2)} \ , \tag{12}$$

which is very suitable for applying the Borel transformation, since

$$\hat{B}_{p^2}(M^2)\, e^{-\alpha p^2} = \delta(1-\alpha M^2) \ . \tag{13}$$

The analytical expression for the Wilson coefficient of the gluon condensate operator C_{G^2} is quite lengthy and for this reason it is presented in the appendix.

It should be noted that Borel transformed Wilson coefficient of the gluon condensate contribution in the three point sum rules with arbitrary mass, which appears in the study of the form factors for the vector and axial vector current transitions of the semileptonic $B_c \to J/\psi \ell \nu$ decay, was investigated in detail in [16].

We now turn our attention to the computation of the physical part of the sum rules. Assuming that the spectral density is well convergent, the physical spectral density is saturated by the lowest lying hadronic states plus a continuum starting at some effective thresholds $s_0,\ s_0'$

$$\rho_{\mu\alpha}^{phy}(s,s') = \rho_{\mu\alpha}^{res}(s,s') + \theta(s-s_0)\theta(s'-s_0')\rho_{\mu\alpha}^{cont}(s,s') \ , \tag{14}$$

where

$$\begin{aligned} \rho_{\mu\alpha}^{res} &= \langle 0 \left| \bar{c} i \gamma_5 b \right| B_c \rangle \langle B_c \left| \bar{u} i \sigma_{\mu\nu} q^\nu c \right| B_u^* \rangle \left\langle B_u^* \left| \bar{b} \gamma_\alpha u \right| 0 \right\rangle \\ &\times \ (2\pi)^2 \delta(s - M_{B_c}^2)\delta(s'-M_{B_u^*}^2) \ , \end{aligned} \tag{15}$$

and ρ^{cont} corresponds to the continuum contribution. The matrix elements in (15) are defined in the following way:

$$\begin{aligned} \langle 0 \left| \bar{c} i \gamma_5 b \right| B_c \rangle &= \frac{f_{B_c} m_{B_c}^2}{m_b + m_c} \ , \\ \left\langle B_u^* \left| \bar{b} \gamma_\alpha u \right| 0 \right\rangle &= f_{B_u^*} m_{B_u^*} \varepsilon_\alpha^* \ . \end{aligned}$$

Selecting the structure $i\epsilon_{\mu\nu\alpha\beta} p^\alpha p'^\beta$ for ρ^{res}, we have

$$\rho^{res} = \frac{f_{B_c} m_{B_c}^2}{m_b + m_c} f_{B_u^*} m_{B_u^*} F_1(0) (2\pi)^2 \delta(s - M_{B_c}^2)\delta(s'-M_{B_u^*}^2) \ .$$

The continuum contribution is modeled as a perturbative contribution starting from thresholds s_0 and s_0'. Equating (15) to the theoretical part contribution (7) and performing double

Borel transformations with respect to the parameters p^2 and p'^2, we finally get the following sum rule for the transition form factor:

$$
\begin{aligned}
F_1(0) &= \frac{(m_b+m_c)}{f_{B_u^*} m_{B_u^*} f_{B_c} m_{B_c}^2} e^{m_{B_c}^2/M_1^2} e^{m_{B_u^*}^2/M_2^2} \\
&\times \left\{ -\frac{1}{4\pi^2} \int ds ds' \rho(s,s') e^{-s/M_1^2} e^{-s'/M_2^2} + M_1^2 M_2^2 \langle \frac{\alpha_s}{\pi} G^2 \rangle C_{G^2} \right\} , \qquad (16)
\end{aligned}
$$

where C_{G^2} is the Wilson coefficient of the gluon condensate contribution. This expression is the final result for the transition form factor evaluated at $q^2 = 0$.

For the numerical analysis we have used the following values of the input parameters that enter into sum rules (16): $f_{B_c} = 385\ MeV$ [17], $f_{B_u^*} = 160\ MeV$ [18], $m_c = 1.4\ GeV$, $\langle \alpha_s G^2/\pi \rangle = 0.012\ GeV^4$ [10], $s_0 = 50\ GeV^2$ and $s_0' = 35\ GeV^2$. As is obvious, Eq. (16) involves two independent Borel parameters M_1^2, M_2^2, and then the main problem is finding the region where the dependence of these parameters is weak and at the same time power corrections and the continuum remains under control.

In Fig. (2) we present the dependence of the transition form factor $F_1(0)$ on M_1^2 and M_2^2. Numerical calculations show that the best stability for the form factor $F_1(0)$ is achieved for $15\ GeV^2 \leq M_1^2 \leq 20\ GeV^2$ and $8\ GeV^2 \leq M_2^2 \leq 12\ GeV^2$. Our final result for the form factor is

$$
F_1(0) = (0.9 \pm 0.1) \ .
$$

For a comparison, we note that the IGSW model's prediction for this transition form factor is $F_1(0) = 0.48$ [3]. Therefore the branching ratio in our case is approximately four times larger compared to that predicted in [3]. It should be stressed again that in the present work only the short distance contribution to the $B_c \to B_u^* \gamma$ decay is considered.

Using this result we observe that the short distance contribution to the branching ratio is of the order of $\sim 2 \times 10^{-8}$, which can be quite detectable at future LHC. Moreover our result show that the short distance contribution in our approach is comparable or larger than the long distance contribution calculated in [3]

$$
\mathcal{B}\left(B_c \to B_u^* \gamma\right) = \left(7.5^{+7.7}_{-4.3}\right) \times 10^{-9},
$$

and our result proves that there is indeed real possibility for probing the $c \to u\gamma$ decay via the beauty conserving $B_c \to B_u^* \gamma$ decay.

As the final remark we would like to note that the approach presented in this work is applicable for calculating the short distance contributions to the branching ratio of the $B_s \to B_d^* \gamma$ and $B_c \to D_s^* \gamma$ decays.

Appendix : **The gluon condensate contribution**

In this section we will present the explicit expressions for the Wilson coefficients C_{G^2} for each diagram, after the Borel transformation with respect to p^2 and p'^2, which are presented in Fig. (1).

$$
\begin{aligned}
(C_{G^2})_1 &= 96 m_c \Big\{ \Big[m_b \Big(I_0[1,3,1] + m_c^2 I_0[1,4,1] + I_1[1,3,1] + m_c^2 I_1[1,4,1] \\
&+ I_2[1,3,1] - m_c^2 I_2[1,4,1] \Big) \Big] + m_c \Big(I_1[1,3,1] + m_c^2 I_1[1,4,1] + 2 I_3[1,4,1] \Big) \Big\} ,
\end{aligned} \tag{A.1}
$$

$$
\begin{aligned}
(C_{G^2})_2 &= 16 \Big\{ 2 I_0[1,1,2] + 2 m_c m_b I_0[1,1,3] + 2 I_0[2,1,1] + 3 m_c m_b I_0[2,1,2] \\
&+ 4 m_b^2 I_0[2,1,2] + 4 m_c m_b^3 I_0[2,1,3] + 2 m_c m_b I_0[3,1,1] + 2 m_b^2 I_0[3,1,1] \\
&+ 6 m_c m_b^3 I_0[3,1,2] + 2 m_b^4 I_0[3,1,2] + 2 m_c m_b^5 I_0[3,1,3] + 2 I_1[1,1,2] \\
&- 2 m_c^2 I_1[1,1,3] + 2 m_c m_b I_1[1,1,3] + 2 I_1[2,1,1] - m_c^2 I_1[2,1,2] \\
&+ m_c m_b I_1[2,1,2] + 4 m_b^2 I_1[2,1,2] - 4 m_c^2 m_b^2 I_1[2,1,3] + 4 m_c m_b^3 I_1[2,1,3] \\
&- 2 m_c^2 I_1[3,1,1] + 2 m_b^2 I_1[3,1,1] - 6 m_c^2 m_b^2 I_1[3,1,2] + 4 m_c m_b^3 I_1[3,1,2] \\
&+ 2 m_b^4 I_1[3,1,2] - 2 m_c^2 m_b^4 I_1[3,1,3] + 2 m_c m_b^5 I_1[3,1,3] + 2 m_c m_b I_2[1,1,3] \\
&+ I_2[2,1,1] + m_c m_b I_2[2,1,2] + 4 m_c m_b^3 I_2[2,1,3] - 4 m_b^2 I_2[3,1,1] \\
&+ 4 m_c m_b^3 I_2[3,1,2] + 2 m_c m_b^5 I_2[3,1,3] - 4 I_3[1,1,3] - 4 I_3[2,1,2] \\
&- 8 m_b^2 I_3[2,1,3] - 8 I_3[3,1,1] - 16 m_b^2 I_3[3,1,2] - 4 m_b^4 I_3[3,1,3] \Big\} \\
&- 32 M_2^2 \frac{d}{dM_2^2} \Big\{ M_2^2 \Big[I_0[2,1,2] + 2 m_c m_b I_0[2,1,3] + 2 m_c m_b I_0[3,1,2] \\
&+ m_b^2 I_0[3,1,2] + 2 m_c m_b^3 I_0[3,1,3] + I_1[2,1,2] - 2 m_c^2 I_1[2,1,3] + 2 m_c m_b I_1[2,1,3] \\
&- 2 m_c^2 I_1[3,1,2] + m_c m_b I_1[3,1,2] + m_b^2 I_1[3,1,2] - 2 m_c^2 m_b^2 I_1[3,1,3] \\
&+ 2 m_c m_b^3 I_1[3,1,3] - I_2[2,1,2] + 2 m_c m_b I_2[2,1,3] - 2 I_2[3,1,1] \\
&+ m_c m_b I_2[3,1,2] - m_b^2 I_2[3,1,2] + 2 m_c m_b^3 I_2[3,1,3] - 4 I_3[2,1,3] \\
&- 6 I_3[3,1,2] - 4 m_b^2 I_3[3,1,3] \Big] \Big\} \\
&- 32 M_2^4 \left(\frac{d^2}{dM_2^2} \right)^2 \Big\{ M_2^4 \Big[m_c^2 I_1[3,1,3] + 2 I_2[3,1,2] - m_c m_b \Big(I_0[3,1,3] + I_1[3,1,3] \\
&+ I_2[3,1,3] \Big) + 2 I_3[3,1,3] \Big] \Big\} ,
\end{aligned} \tag{A.2}
$$

$$
\begin{aligned}
(C_{G^2})_3 &= 96 m_b \Big\{ - \Big(m_c^2 m_b I_1[4,1,1] \Big) + m_c \Big(I_0[3,1,1] + m_b^2 I_0[4,1,1] \\
&+ I_1[3,1,1] + m_b^2 I_1[4,1,1] + I_2[3,1,1] + m_b^2 I_2[4,1,1] \Big) - 2 m_b I_3[4,1,1] \Big\} ,
\end{aligned} \tag{A.3}
$$

$$
\begin{aligned}
(C_{G^2})_4 = & -32m_c m_b\Big\{I_0[2,1,2]+I_0[3,1,1]+m_b^2 I_0[3,1,2]+I_1[2,1,2]+m_b^2 I_1[3,1,2] \\
& +I_2[2,1,2]+m_b^2 I_2[3,1,2]-4I_3[3,1,2]-M_2^2\frac{d}{dM^2}\Big[M_2^2\Big(I_0[3,1,2]+I_1[3,1,2] \\
& +I_2[3,1,2]\Big)\Big]\Big\}+16\Big\{m_c^2 I_1[2,1,2]+I_2[2,1,1]-m_c m_b\Big(I_0[2,1,2]+I_1[2,1,2] \\
& +I_2[2,1,2]\Big)+4I_3[2,1,2]\Big\}\ ,
\end{aligned} \tag{A.4}
$$

$$
\begin{aligned}
(C_{G^2})_5 = & \ 16\Big\{2I_0[1,1,2]+I_0[1,2,1]+2m_c^2 I_0[1,2,2]+m_c m_b I_0[1,2,2] \\
& +m_b^2 I_0[2,1,2]+m_c m_b I_0[2,2,1]+m_b^2 I_0[2,2,1]+m_c^2 m_b^2 I_0[2,2,2] \\
& +m_c m_b^3 I_0[2,2,2]+I_1[1,1,2]+I_1[1,2,1]+m_c^2 I_1[1,2,2] \\
& +m_c m_b I_1[1,2,2]-I_1[2,1,1]-2m_c m_b I_1[2,1,2]-m_c^2 I_1[2,2,1] \\
& +m_b^2 I_1[2,2,1]-2m_c^3 m_b I_1[2,2,2]+m_c m_b^3 I_1[2,2,2]+I_2[1,1,2] \\
& +I_2[1,2,1]+m_c^2 I_2[1,2,2]+m_c m_b I_2[1,2,2]-2m_c m_b I_2[2,2,1] \\
& +m_b^2 I_2[2,2,1]+m_c m_b^3 I_2[2,2,2]-2I_3[2,1,2]-4I_3[2,2,1] \\
& -2m_c^2 I_3[2,2,2]-4m_c m_b I_3[2,2,2]\Big\} \\
& -16M_1^2\frac{d}{dM_1^2}\Big\{M_1^2\Big[I_2[2,2,1]+2I_3[2,2,2]\Big]\Big\} \\
& +16M_2^2\frac{d}{dM_2^2}\Big\{M_2^2\Big[-m_c m_b I_0[2,2,2]+I_1[2,1,2]+m_c^2 I_1[2,2,2]-m_c m_b I_1[2,2,2] \\
& +I_2[2,1,2]+I_2[2,2,1]+m_c^2 I_2[2,2,2]-m_c m_b I_2[2,2,2]+2I_3[2,2,2]\Big]\Big\} \\
& +16\Big\{I_1[1,1,2]+m_c^2 I_1[1,2,2]+I_2[1,2,1]-m_c m_b\Big(I_0[1,2,2]+I_1[1,2,2] \\
& +I_2[1,2,2]\Big)+2I_3[1,2,2]\Big\}\ ,
\end{aligned} \tag{A.5}
$$

$$
\begin{aligned}
(C_{G^2})_6 = & \ 16\Big\{2I_0[1,2,1]+2I_0[2,1,1]+2m_c^2 I_0[2,2,1] \\
& +6m_c m_b I_0[2,2,1]+2m_b^2 I_0[2,2,1]+2I_1[1,2,1]-5I_1[2,1,1]-5m_c^2 I_1[2,2,1] \\
& +6m_c m_b I_1[2,2,1]+2m_b^2 I_1[2,2,1]+2I_2[1,2,1]-I_2[2,1,1] \\
& -m_c^2 I_2[2,2,1]+6m_c m_b I_2[2,2,1]+2m_b^2 I_2[2,2,1]-14I_3[2,2,1]\Big\} \\
& -32M_1^2\frac{d}{dM_1^2}\Big\{M_1^2\Big[I_0[2,2,1]+I_1[2,2,1]+I_2[2,2,1]\Big]\Big\}\ ,
\end{aligned} \tag{A.6}
$$

where the subscripts in the Wilson coefficients C_{G^2} denote the corresponding diagrams in Fig. (1). In calculating the gluon condensate contribution, we need integrals of the following types:

$$
I_0[a,b,c]=\int\frac{d^4k}{(2\pi)^4}\frac{1}{\left[k^2-m_b^2\right]^a\left[(p+k)^2-m_c^2\right]^b\left[(p'+k)^2\right]^c}\ , \tag{A.7}
$$

$$I_\mu[a,b,c] = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{[k^2-m_b^2]^a\,[(p+k)^2-m_c^2]^b\,[(p'+k)^2]^c} \ , \tag{A.8}$$

$$I_{\mu\nu}[a,b,c] = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{[k^2-m_b^2]^a\,[(p+k)^2-m_c^2]^b\,[(p'+k)^2]^c} \ . \tag{A.9}$$

The integrals I_μ and $I_{\mu\nu}$ can be written in the following form

$$\begin{aligned} I_\mu &= I_1 p_\mu + I_2 p'_\mu \ , \\ I_{\mu\nu} &= I_3 g_{\mu\nu} + I_4 p_\mu p_\nu + I_5 p'_\mu p'_\nu + I_6 p_\mu p'_\nu + I_7 p'_\mu p_\nu \ . \end{aligned} \tag{A.10}$$

It should be noted that only the $g_{\mu\nu}$ term in $I_{\mu\nu}$ gives contribution to the $\epsilon_{\mu\nu\alpha\beta}$ structure which we need in our analysis.

After double Borel transformations with respect to the variables p^2 and p'^2, the explicit forms of the integrals $I_0[a,b,c]$, $I_1[a,b,c]$, $I_2[a,b,c]$ and $I_3[a,b,c]$ are as follows (see also [16])

$$I_0[a,b,c] = \frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-4, 1-c-b) \ , \tag{A.11}$$

$$I_1[a,b,c] = \frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{2-a-b}(M_2^2)^{3-a-c}\,\mathcal{U}_0(a+b+c-5, 1-c-b) \ , \tag{A.12}$$

$$I_2[a,b,c] = \frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-5, 1-c-b) \ , \tag{A.13}$$

$$I_3[a,b,c] = \frac{(-1)^{a+b+c+1}}{32\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} (M_1^2)^{3-a-b}(M_2^2)^{3-a-c}\,\mathcal{U}_0(a+b+c-6, 2-c-b) \ . \tag{A.14}$$

The function $\mathcal{U}_0(\alpha,\beta)$ is given by the following expression

$$\mathcal{U}_0(\alpha,\beta) = \int_0^\infty dy (y + M_1^2 + M_2^2)^\alpha y^\beta \, exp\left[-\frac{B_{-1}}{y} - B_0 - B_1 y\right] \ , \tag{A.15}$$

where

$$\begin{aligned} B_{-1} &= \frac{m_c^2}{M_1^2}\left[M_1^2 + M_2^2\right] \ , \\ B_0 &= \frac{1}{M_1^2 M_2^2}\left[M_1^2 m_b^2 + M_2^2(m_b^2 + m_c^2)\right] \ , \\ B_1 &= \frac{m_b^2}{M_1^2 M_2^2} \ . \end{aligned} \tag{A.16}$$

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Figure captions

Fig. 1 Gluon condensate contribution diagrams to the $B_c \to B_u^* \gamma$ decay. In this figure the dashed line represents the soft gluon line, c, u, b identify the quark lines, p and p' are the four–momenta of the incoming B_c and outgoing B_u^* mesons, respectively, and q is the four–momentum of the outgoing photon.

Fig. 2 The dependence of the transition form factor $F_1(0)$ on the Borel parameters M_1^2 and M_2^2.

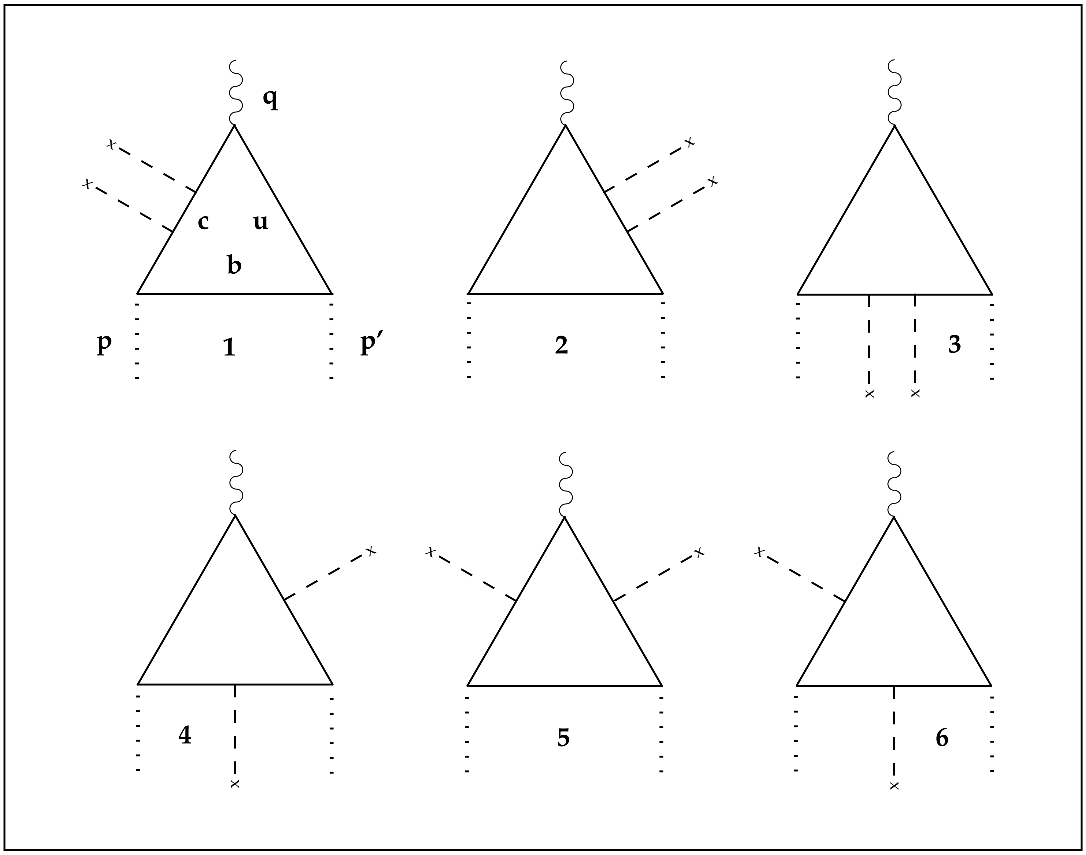

Figure 1:

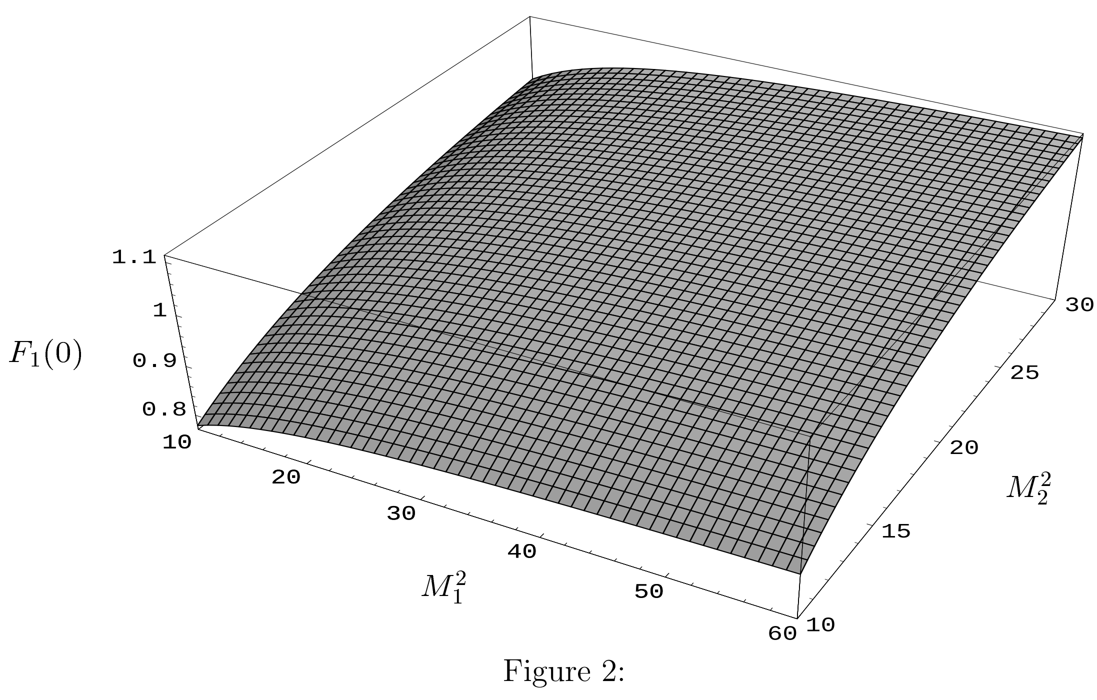

Figure 2: